Resonant Frequencies of the Axial Symmetric Modes in a Dielectric Resonator

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ABSTRACT

Dielectric resonators have been proven possible for a long time. However, they had not been popular in the past due to the absence of temperature stable and low loss materials. The recent advent of low loss, temperature stable materials has made them useful in a number of microwave circuit applications. The analysis of such a resonator in the past has relied on approximate methods. We shall present a rigorous field analysis of the circular dielectric resonator embedded in an inhomogeneous medium. The analysis is via a numerical mode matching method, whereby the problem of finding the modes of the circular dielectric cylinder is cast into a conventional eigenvalue problem which could be solved rapidly on the computer. This method bypasses the need to use Hankel and Bessel functions, which could be time consuming to evaluate. The scattering of the field off the ends of the resonator are characterized by reflection operators. The resonant frequencies of the resonator could be easily found by requiring the phase coherence of the wave after reflection off the two ends of the resonator.

I. Introduction

Dielectric resonators have become quite popular recently due to the advent of low loss, temperature stable materials[1]. They have been used as filters, frequency stabilizers and feedback circuits in various microwave circuits applications. Furthermore, they could be easily integrated with microwave integrated circuits. The use of low loss, high dielectric constant material has made the Q of such a resonator considerably higher, and the size smaller compared to conventional metallic cavity resonators. However, the rigorous analysis of the dielectric resonator is behind its applications. The earliest method to estimate the resonant frequency of such a resonator were made with the magnetic wall model[2], while many recent analyses of the dielectric resonator still make use of approximate methods[3-6]. Recently, there has been some work where the analysis of the resonant frequency of such a resonator were performed more accurately[7-10]. The rigorous analyses in the past have restrictive geometry. In this paper, we present an analysis which is rigorous, but the geometry has a certain degree of versatility. A rigorous field analysis could predict the resonant frequencies of a low dielectric constant resonator. A low dielectric constant resonator could be useful at a high frequencies where the limitation in manufacturing tolerance may dictate the use of a larger resonator.

II. Formulation

The equation governing the axial symmetric azimuthal electric field in a general circular dielectric cylinder is given by

$$\left(\rho\mu\frac{\partial}{\partial\rho}\frac{1}{\rho\mu}\frac{\partial}{\partial\rho}+\frac{\partial^2}{\partial z^2}+\omega^2\mu\epsilon\right)\rho E_\phi=0. \tag{1}$$

Since the TM solution can be obtained from the TE solution by invoking duality principle, we need only to consider the TE case here. In the above, μ and ϵ are arbitrary functions of ρ. To find the modes or eigenfunctions of the cylinder, we let

$$\rho E_\phi(\rho,z)=\sum_\alpha f_\alpha(\rho)e^{ik_{\alpha z}z}a_\alpha, \tag{2}$$

where a_α is a constant independent of ρ and z. It follows that we can let

$$f_\alpha(\rho)=\sum_{n=1}^{N}b_{\alpha n}g_n(\rho), \tag{3}$$

where $g_n(\rho)$ are some known basis functions and $b_{\alpha n}$ are the unknowns to be sought. Substituting (3) into (2) and later into (1), we obtain for each α that

$$\sum_{n=1}^{N}b_{\alpha n}\left(\rho\mu\frac{\partial}{\partial\rho}\frac{1}{\rho\mu}\frac{\partial}{\partial\rho}+\omega^2\mu\epsilon-k_{\alpha z}^2\right)g_n(\rho)=0. \tag{4}$$

We can multiply the above by $(\rho\mu)^{-1}g_m(\rho)$ and integrate from zero to infinity to eliminate the ρ dependence in the equation. Hence, the above becomes

$$\sum_{n=1}^{N}b_{\alpha n}\left(B_{nm}-k_{\alpha z}^2G_{nm}\right)=0 \tag{5}$$

where

$$B_{nm}=-\int_0^\infty d\rho\frac{1}{\rho\mu}g'_n(\rho)g'_m(\rho)$$

$$+\omega^2\int_0^\infty d\rho\frac{\epsilon}{\rho}g_n(\rho)g_m(\rho) \tag{6}$$

and

$$G_{nm}=\int_0^\infty d\rho\frac{1}{\rho\mu}g_n(\rho)g_m(\rho) \tag{7}$$

Equation (5) could be solved for the eigenvalues and eigenvectors easily. From that we can easily construct the eigenfunctions of equation (2). With these eigenfunctions available, we can write down easily the solutions in region 2 [see Figure 1] of the inhomogeneity.

$$\rho E_{2\phi}=\bar{J}_2^t(\rho)\cdot\left(e^{i\bar{K}_{2z}z}+e^{-i\bar{K}_{2z}z}\cdot\bar{R}_{21}\right)\cdot\bar{a} \tag{8}$$

and the field in region 1 could be written as

$$\rho E_{1\phi}=\bar{J}_1^t(\rho)\cdot e^{i\bar{K}_{1z}z}\cdot\bar{T}_{12}\cdot\bar{a} \tag{9}$$

where the variables are more properly defined in reference [11]. The boundary conditions could be invoked to find the reflection and transmission operators. A similar expression as in region 1 can be written in region 3. By requiring the coherence of the upgoing and downgoing waves in region 2, the resonance of the dielectric resonator could be written as

$$\det\left(\bar{I}-\bar{R}_{21}\cdot e^{i\bar{K}_{2z}d}\cdot\bar{R}_{23}\cdot e^{i\bar{K}_{2z}d}\right)=0. \tag{10}$$

The above equation could be solved to find the resonance frequencies of the resonators, as the equation could only be satisfied at a some frequencies which are the resonant frequencies of the structure.

III. Numerical Result

We can compute the resonant frequencies of the dielectric resonator via the above approach. Figure 2 shows the normalized resonant frequencies as a function of the contrast for the dielectric resonator with $d/a=1$. $k_r a$ is the normalized resonant frequency. $k_r a$ tends to be larger for higher contrast because the mode is better trapped inside the resonator. Figure 3 shows the case for $d/a=2$. In this case the asymptotic tends to be lower, implying a lower resonant frequency because the z-variation of the field is slower in this case. Figure 4 shows the resonator embedded in a substrate. This is the model which could be study by our program, but not by previous rigorous analyses.

References

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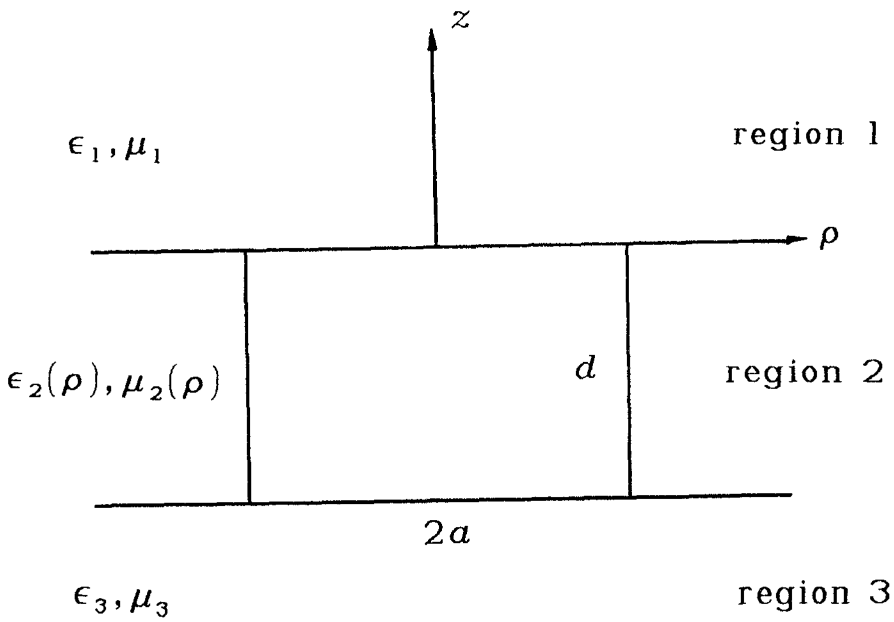

Figure 1. Geometry of the problem

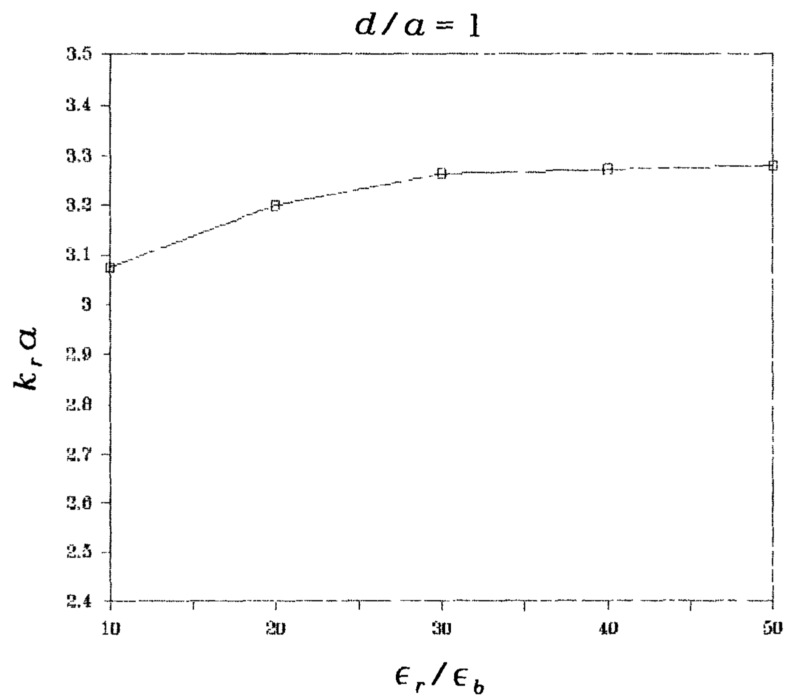

Figure 2. Normalized frequency versus the contrast of a dielectric resonator, d/a=1.

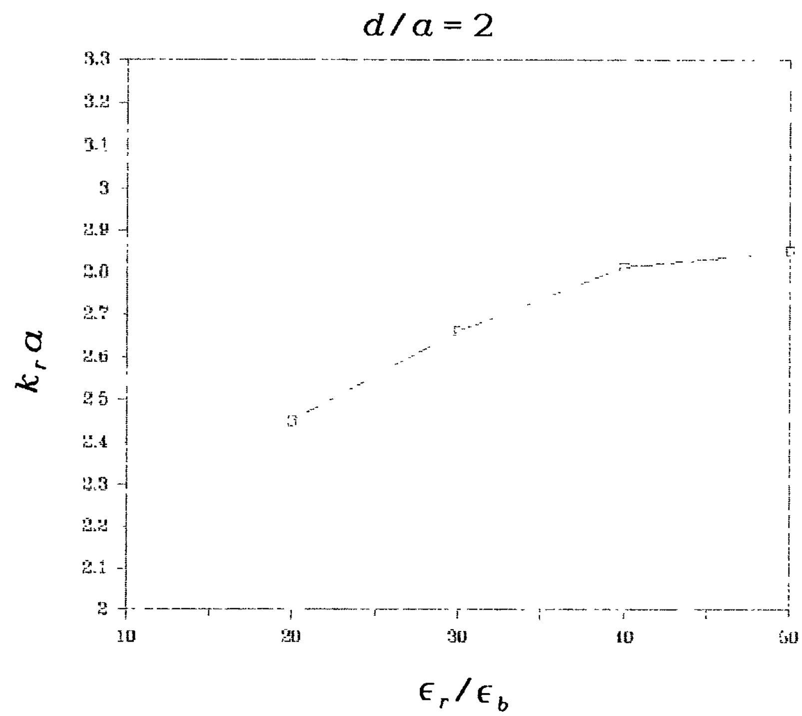

Figure 3. Normalized frequency versus the contrast of a dielectric resonator, d/a=2.

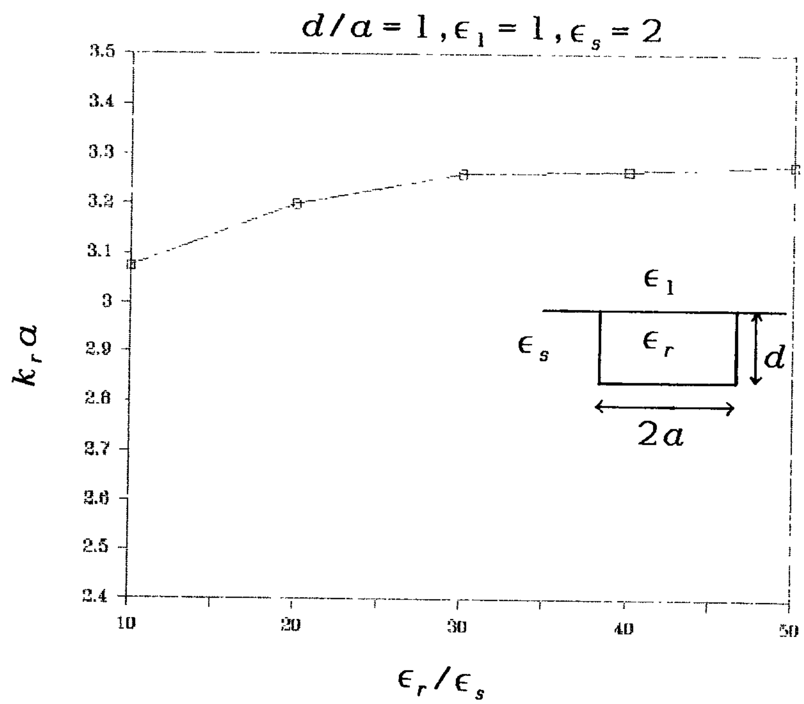

Figure 4. Normalized frequency versus the contrast of a dielectric resonator embedded in a substrate.